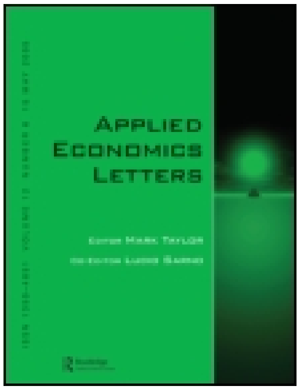

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The strange persistence of consumer surplus

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The strange persistence of consumer surplus

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Despite its abandonment in theoretical work, a literature search shows that variation in consumer surplus (VCS) is the overwhelming choice in applied work – not compensating variation (CV) or equivalent variation (EV). How can this be explained? Besides the obvious ease of computation, there are three good reasons for the persistence of VCS. (1) The Willig bounds on VCS usually give close upper and lower bounds on CV and EV, respectively, and are thus *conservative* in the estimation of EV. (2) Without integrability, all three measures are inaccurate. Common quasi-linear utility assumptions for VCS, however, imply integrability. (3) Even with integrability, the expected values of highly nonlinear CV and EV measures cannot be determined by substituting prices or quantities into the estimated equations; simulations are required. Thus, VCS is not only simpler, it may also be more accurate.

Keywords: Willig bounds; integrability; non-linearity; simulation

JEL Classification: D60; C13

I. Introduction

This article seeks to explain a puzzle. When considering consumers' evaluations of price change, economic theory requires measures derived from explicit utility functions. Equivalent variation (EV) and compensating variation (CV) are so derived; variation in consumer surplus (VCS) is not. Explicit recommendations to abandon VCS in favour of *computable forms* of EV and CV are advanced in classic articles by Hausman (1981), King (1983) and Vartia (1983).

These articles, cited in most reviews of empirical welfare economics, have done little to influence applied work. The overwhelming bulk of research since 1981 has continued to ignore EV and CV in favour of VCS. Consider searches for the string 'consumer surplus', with and without the strings 'equivalent variation' or 'compensating variation'. Searches were performed in *EconLit* and the 'Business, Administration, Finance, and Economics' category of *Google Scholar*.

Peer-reviewed articles in the *Journal of Economic Literature* (*EconLit*) database show greater consumer surplus 'only' usage than the more inclusive *Google Scholar*. But while *EconLit* shows EV and CV as making no progress over 30 years, *Google Scholar* shows them as losing ground (Table 1). Consumer surplus remains the overwhelming choice in applied work, as for example in World Bank projects (Peskin, 2006, p. 1).

It is hard to believe that the majority of economic authors, referees and editors do not *know* EV and CV are the theoretically preferred measures. It is only slightly less incredible that they know but are too lazy to do the calculations. How then can we explain this 'indefensible' (or at least, undefended) preference for VCS measures?

It is the thesis of this article that VCS has three important advantages over the exact measures, besides the obvious ease of computation. Although each is mentioned in the literature, they are never discussed together, so far as we know. Considering

Table 1. Relative frequency (in %) of consumer welfare measure terms found in *Google Scholar* and the *Journal of Economic Literature, Pre and Post 1982*

	Before 1982		1982 or Later	
Article I.	Number	Relative frequency	Number	Relative frequency
Search String in Google Scholar				
Consumer('s, s') surplus	5420	100	40 220	100
Consumer('s, s') surplus[a]	2083	38	33 420	83
Search string in EconLit				
Consumer('s, s') Surplus	61	100	1067	100
Consumer('s, s') Surplus[a]	60	98	1044	98

Sources: *Google Scholar*, 12 January 2012; *EconLit*, 'peer-reviewed journals,' 12 January 2012.
Note: [a]Indicates appearance of consumer surplus terms *without* the use of either equivalent or compensating variation. This number must be less than or equal to that in the line above.

them together makes it clear that VCS is not only a simpler, but often a more accurate choice.

In decreasing order of their incidence in the literature, the three advantages are as follows:

(1) *Upper and Lower Bounds*: The Willig (1976) conditions usually provide tight upper and lower bound on CV and EV, respectively: $\sum_i \text{CV}_i < \sum_i \text{VCS}_i < \sum_i \text{EV}_i$. The last inequality makes the VCS measure *conservative*.
(2) *Integrability Restrictions*: Neither VCS, CV, nor EV is accurate without integrability. The common assumption of quasi-linear utility, however, achieves both integrability and a simple measure of VCS.
(3) *Nonlinear Simulations*: Even if integrability is imposed, the nonlinearity of demand means that estimated parameters cannot be directly substituted into expenditure functions. Linear VCS measures avoid this inaccuracy.

The logic behind these issues will be developed in Sections II–IV.

II. Upper and Lower Bounds

Varian (1992, p. 168) notes the well-known relationship between these measures. For any consumer i and a normal good, regardless of the sign of the values,

$$\text{CV}_i < \text{VCS}_i < \text{EV}_i \tag{1}$$

The implication of Equation 1 is that VCS is a *conservative estimate of the welfare change measured by EV*. For a fall in prices, ($p_0 > p_1$), a positive-valued VCS(+) is conservative in the sense of showing *less benefit* than EV(+). For a rise in prices ($p_1 > p_0$), a negative-valued VCS(−) is conservative in the sense of showing *more harm* than EV(−) or EV(+).

This conservative sense must also hold in aggregate. Since Equation 1 holds regardless of sign, then summing over i, the following inequality shows that *net* VCS over all individuals must lie between the other measures:

$$\begin{aligned} \sum i\,\text{CV}_i(+) + \text{CV}_i(-) < \sum i\text{VCS}_i(+) \\ + \text{VCS}_i(-) < \sum i\text{EV}_i(+) + \text{EV}_i(-) \end{aligned} \tag{2}$$

If the null hypothesis is that most projects have an EV less than X, then the VCS criterion is again conservative in showing *no Type I errors*, that is it accepts only values high enough to show $\sum_i \text{EV} > \sum_i \text{VCS} > X$. It will, however, reject projects that show $\sum_i \text{EV} > X > \sum_i \text{VCS}$, thus allowing *Type II* errors.

If one wishes to reverse the importance of Type I and Type II errors, then the null hypothesis should be that most projects have a CV less than X. Now a VCS criterion can make Type I errors, since it wrongly rejects the null when $\sum_i \text{CV} < X < \sum_i \text{VCS}$. By the same token, it can now never make a Type II error, since $\sum_i \text{CV} < \sum_i \text{VCS} < X$.

If we are considering a single price change for a normal good, the distance between VCS and these bounds is limited by the well-known Willig (1976) conditions:

$$\begin{aligned} 0 \le (\text{VCS} - \text{CV})/|\text{VCS}| \le \varepsilon^* \left(\frac{1}{2} * |\text{VCS}|\right) \Big/ Y \\ 0 \le (\text{EV} - \text{VCS})/|\text{VCS}| \le \varepsilon^* \left(\frac{1}{2} * |\text{VCS}|\right) \Big/ Y \end{aligned} \tag{3}$$

where ε is the largest value for income elasticity calculated in the region estimated, and Y is the income. These Willig bounds apply only if the surplus to income ratio, $|\mathrm{VCS}|/Y$, is less than or equal to 90% and the entire right hand side of Equation 3 is less than or equal to 5% (Willig, 1976, p. 596). The income elasticity term, ε, is likely to be not far from unity for most 'policy relevant' goods, that is anything other than real luxuries. Surveys of the estimates of income elasticity for private services such as medical care (Blanciforti, 1982; Henderson, 2009, p. 164) or education (Yang, 1998; Gradstein *et al*., 2005, p. 50) generally show values well below 2. Therefore, the 5% condition of Equation 3 will generally be governed by the ratio $|\mathrm{VCS}|/Y$.

III. The Integrability Problem

In their guide to welfare economics and empirical policy, Just *et al*. (2004, p. 176) outline the conditions necessary for Marshallian demand functions to be integrated into expenditure functions (see also Deaton and Muelbauer, 1980) These conditions are that consumption is constrained by income, that demand is homothetic of degree zero in prices and income (i.e., no 'money illusion') and that the Slutsky substitution matrix is symmetric and negative semi-definite.

To ensure proper signs on coefficients, these restrictions must be in place before estimation. The practical meaning of these restrictions may be questioned, and their statistical credibility may not survive hypothesis testing. Most common specifications of demand, linear or log-linear, do not meet integrability (LaFrance, 1985, 1986). The assumption of quasi-linear utility, however, which assumes that demand for the good in question is independent of income, does assure integrability and is common in applied work. Varian notes (2010, p. 258) that the errors from estimating more complicated demand functions may outweigh those from imposing this restriction.

Failure to impose integrability means that *neither* the derived CV and EV measures from Marshallian demand by Hausman (1981) and Vartia (1983), *nor* their 5% bounds on VCS as calculated by Willig (1976) are generally accurate; that is when there is more than one price change, evaluations cannot be consistently ranked by either the VCS, CV or EV criterion (Just and Gilligan, 1998; Just *et al*., 2004).

Thus, many applied researchers stick with these simpler forms and use VCS. In so doing, they are avoiding another serious problem – one arising from the very nonlinear forms usually implied by integrability restrictions themselves (Morey, 2002).

IV. Nonlinear Estimation

In dealing with highly nonlinear forms – like a properly restricted demand function and its integral of expenditure functions – one cannot merely plug an estimated parameter from the former into the latter. As Edward Morey notes in his an insightful survey of expenditure measures under nonlinearity (2002, p. 28): the expected value of a function is not the function of its expected value:

$$\mathrm{E}[f(x)] \neq f(\mathrm{E}[x]).$$

Take, for example, $f(x) = x^2$. Say the values of 4, 8, 16 or 32 have equal probability, so that their expected value is 15. The expected value of the function $\mathbf{E}[f(x)]$ is $0.25[4^2 + 8^2 + 16^2 + 32^2] = 340$, while the function of the expected value, $f(\mathbf{E}[x])$, is $15^2 = 225$. Most estimates use the simpler but wrong direct substitution method. A widely cited survey, for example, advises that 'Once the parameters of this system of demands are estimated, the compensating and equivalent variations of any price-income change can be calculated … after substituting…' (Just *et al*., 2004, p. 179)

The randomness problem arises from two sources: First, of course, there will usually be variation by sample. But second, even with no variation in sampling, a given individual will vary his/her choices in unpredictable ways. This second source of variation is modelled by the random utility models of McFadden (1981), Small and Rosen (1981) and Hanemann (1999). For present purposes, we need not distinguish these two sources: our key parameters, reappearing in the expenditure function, remain random variables.

The relevant question is not whether deterministic linear treatments of EV or CV are correct – they clearly are not. It is rather whether the linear form of most VCS measures may nonetheless be a better estimate than 'linear parameter substitution' into a nonlinear form. Consider a simple linear expenditure system for two goods q_1 and q_2:

$$\begin{aligned} p_1 q_1(p_1, p_2, m) &= \alpha_1 p_1 + \beta_1 m - (\beta_1 \alpha_1 p_1 + \beta_2 \alpha_2 p_2) \\ p_2 q_2(p_1, p_2, m) &= \alpha_2 p_2 + \beta_2 m - (\beta_1 \alpha_1 p_1 + \beta_2 \alpha_2 p_2) \end{aligned} \tag{4}$$

where the subscripted p is a price, q a quantity, m is money income and the α and β terms are parameters to be estimated. Dividing the first and second equation by p_1 and p_2, respectively, gives the Marshallian demand functions:

$$q_1(p_1,p_2,m) = \alpha_1 + \beta_1 m/p_1 - (\beta_1\alpha_1 + \beta_2\alpha_2 p_2/p_1)$$
$$q_2(p_1,p_2,m) = \alpha_2 + \beta_2 m/p_2 - (\beta_1\alpha_1 p_1/p_2 + \beta_2\alpha_2). \tag{5}$$

Note that Equation 5 is homothetic of degree zero in prices and money, as required for integrability. From the expenditure function, it can be shown that a change in the first price from p_1^0 to p_1^1 defines the expenditure measures

$$\begin{aligned} \text{CV} &= m_1 - \alpha_1 p_1^1 \\ &\quad - (m_0 - \alpha_1 p_1^0 - \alpha_2 p_2^0)(p_1^1/p_1^0)^{\beta_1}(p_2^1/p_2^0)^{\beta_2} \\ \text{EV} &= -m_0 + \alpha 1 p_1^1 \\ &\quad + (m_1 - \alpha 1 p_1^1 - \alpha 2 p_2^1)(p_1^0/p_1^1)^{\beta_1}(p_2^0/p_2^1)^{\beta_2} \end{aligned} \tag{6}$$

while $\text{VCS} = (\ln(p_1^1) - \ln(p_1^0))(\beta_1 m + \beta_2\alpha_2 p_2^0) + (p_1^1 - p_1^0)\alpha_1(1-\beta_1).$

The first two expressions for CV and EV are highly nonlinear. It is convenient but wrong to estimate the parameters of Equation 4 and then substitute them directly into these expressions. The VCS expression in Equation 6, by contrast, is linear in the same form as the expenditure function in Equation 4: once consistent estimates of the joint parameters $\beta_1\alpha_1$ and $\beta_2\alpha_2$ in Equation 4 are obtained, the VCS expression (Equation 6) into which they are substituted is linear in these parameters, and the substitution problem has disappeared.

This problem of nonlinear substitution is not widely discussed in the literature, as Morey (2002) notes. His call for simulation measures is reasonable given the highly nonlinear form of most exact measures. But since most VCS forms are linear, such complications are unnecessary.

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